

Name: _____

Unit 1: Transformations in the Coordinate Plane

KEY STANDARDS

MGSE9–12.G.CO.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

MGSE9–12.G.CO.2 Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

MGSE9–12.G.CO.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

MGSE9–12.G.CO.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

MGSE9–12.G.CO.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Define the following Geometric terms:

point:



LINE:



SEGMENT:



RAY:



Plane:



Collinear:



Plane:



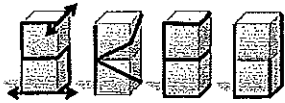
ANGLE:



Parallel Lines:



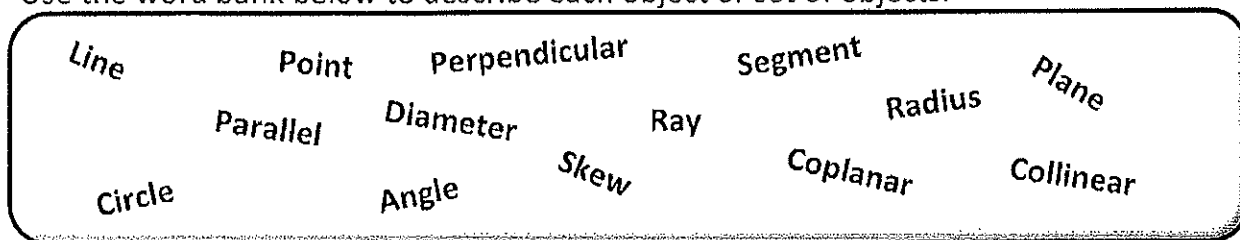
Perpendicular Lines:



Circle:



Use the word bank below to describe each object or set of objects.

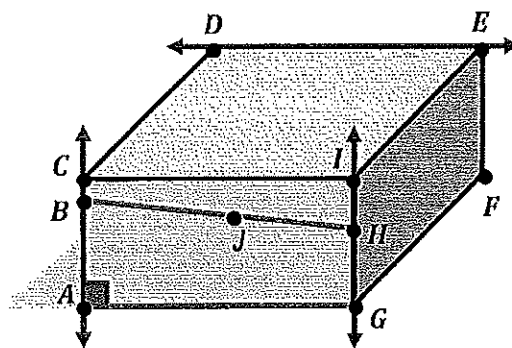


The geometric shape shown in the diagram below is a 3-dimensional rectangular prism.

10. How would you best describe the relationship between the line \overleftrightarrow{AC} and the line \overleftrightarrow{GI} ?

11. How would you best describe the relationship between the line \overleftrightarrow{AC} and the segment \overline{AG} ?

12. How would you best describe the relationship between the line \overleftrightarrow{AC} and the line \overleftrightarrow{DE} ?



13. How would you best describe the set of Point A, Point B, Point J and Point H?

14. How would you best describe the set of Point G, Point H, and Point I?

15. (True or False) Any 3 distinct points are always coplanar.

16. (True or False) Any 2 distinct circles are always coplanar.

17. (True or False) If 2 lines intersect once then the lines are coplanar.

18. (True or False) Two lines that are skew can sometimes intersect.

19. (True or False) An angle and a circle can have more than 3 intersections.

20. (True or False) Two distinct circles can have more than 2 intersections.

21. (True or False) Any given line and point are always coplanar.

Practice A

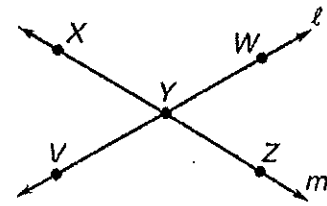
For use with pages 10-16

Draw a sketch and label as needed.

- Three collinear points, A , B , and C .
- \overleftrightarrow{MN} intersecting \overleftrightarrow{PQ} at point R .
- Coplanar points W , X , Y , and Z .
- Opposite rays, \overrightarrow{JK} and \overrightarrow{JC} .

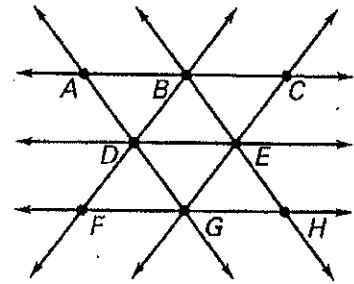
Decide whether the statement is *true* or *false*.

- Point X lies on line m .
- X , Y , and Z are collinear.
- Point W lies on line m .
- X , Y , and Z are coplanar.
- Point V lies on line l .
- V , Y , and X are collinear.
- Point Y lies on line l .
- V , Y , and X are coplanar.



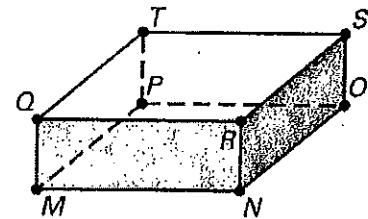
Name a point that is collinear with the given points.

- B and E
- F and H
- D and G
- A and C
- H and E
- G and C
- A and D
- B and C



Name a point that is coplanar with the given points.

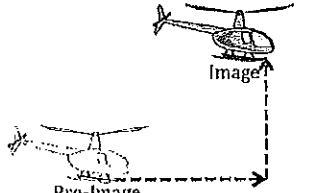
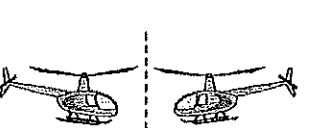
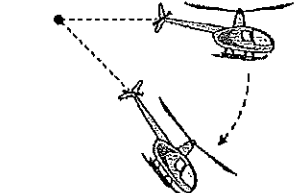
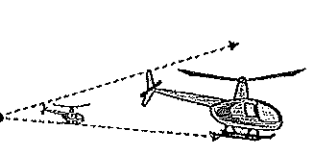
- M , N , and O
- M , N , and R
- T , Q , and M
- T , Q , and R
- T , S , and R
- T , S , and O
- O , S , and R
- O , P , and M



In Exercises 29-34, complete the sentence.

- Collinear points are points that _____.
- Coplanar points are points that _____.
- \overline{XY} consists of the endpoints X and Y and all points on the line \overleftrightarrow{XY} that lie _____.
- \overrightarrow{MN} consists of the initial point M and all points on the line \overleftrightarrow{MN} that lie _____.
- Two rays or segments are collinear if they _____.
- \overrightarrow{PQ} and \overrightarrow{PT} are opposite rays if _____.
- Explain the difference between \overrightarrow{BC} and \overrightarrow{CB} .

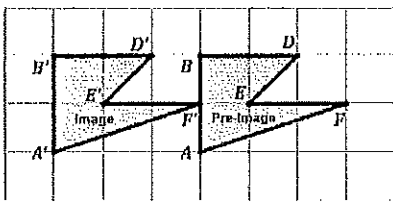
Transformation Types:

Translation ↓ Translation	Reflection ↕ Reflection	Rotation ↻ Rotation	Dilation ••• Dilation
			

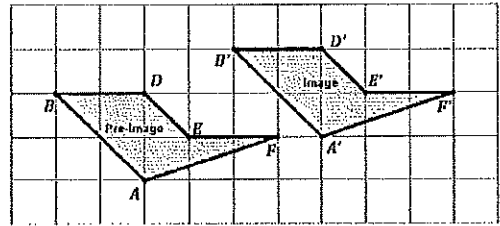
Translations

1. Describe the transformation in rectangular units from the Pre-Image to the Image for each of the following:

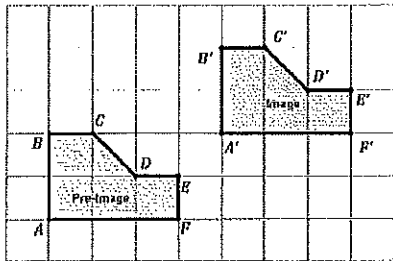
A.



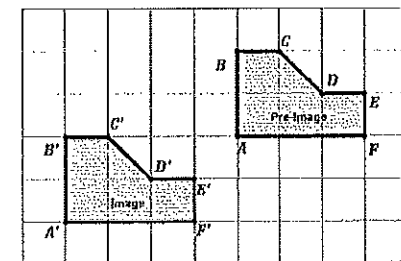
B.



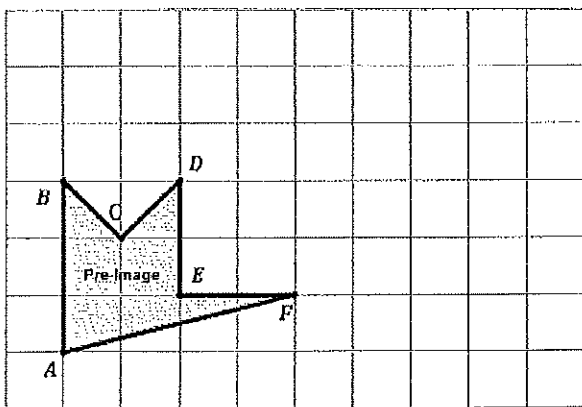
C.



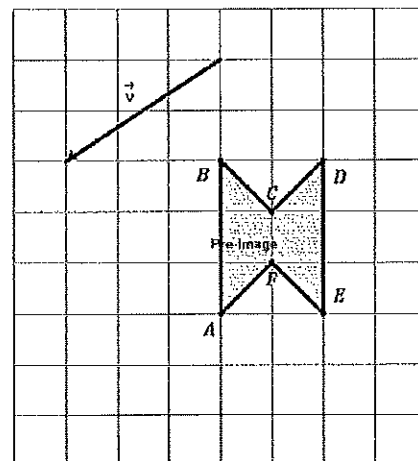
D.



2. Translate the following object right 4 units and up 1 unit. Label each vertex appropriately.

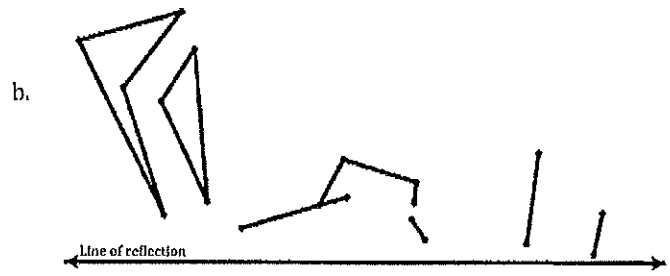
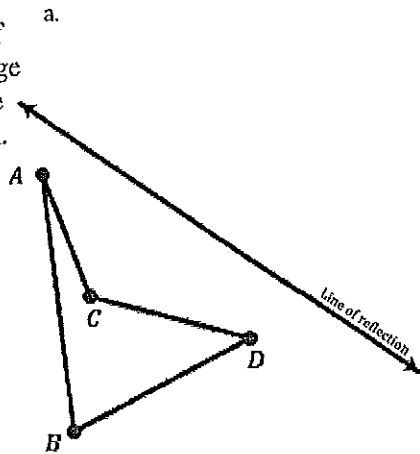


3. Translate the following object by the vector \vec{v} .

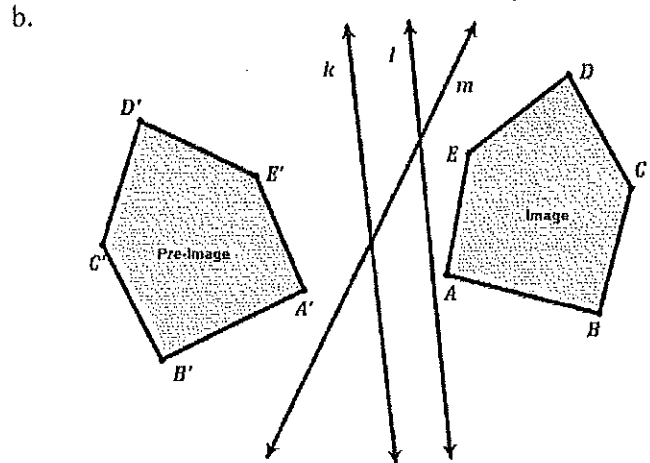
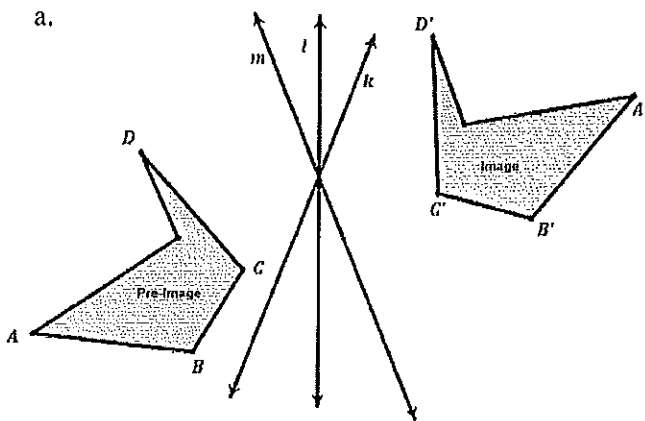


Reflections

4. Create a reflection of the Pre-image over the line of reflection.

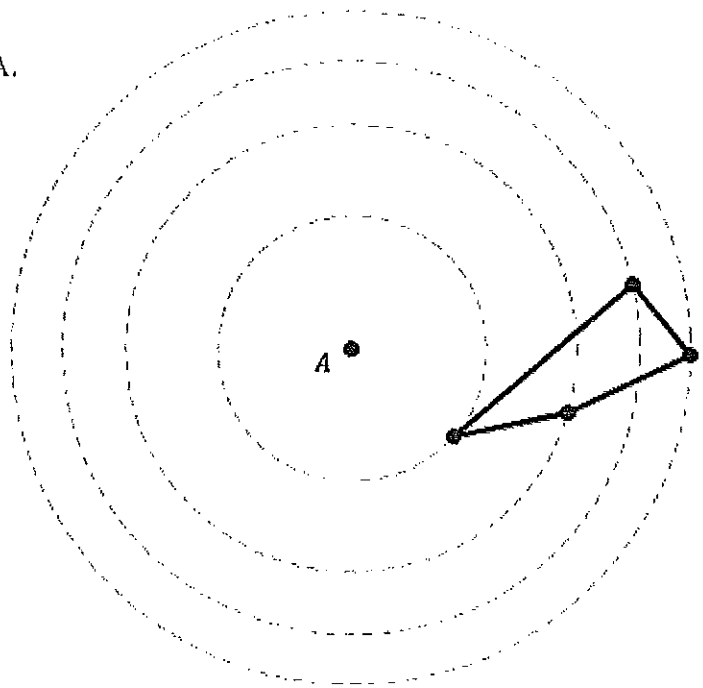


5. Determine which is the correct line of reflection in each diagram between the Image and Pre-Image?



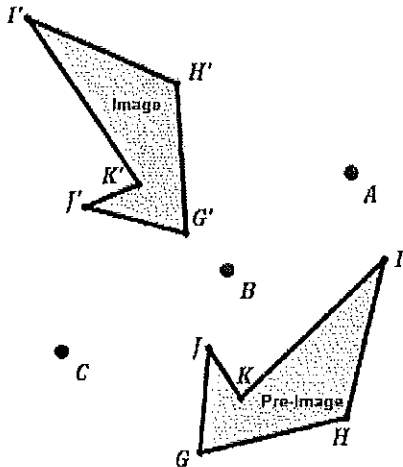
Rotations

6. Rotate the following polygon 110° about the point A.

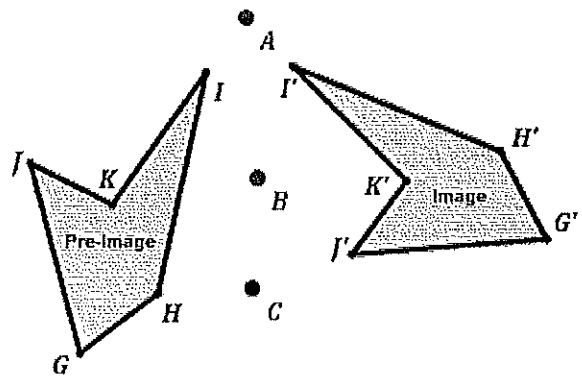


7. Determine which is the correct center if an 80° rotation was used?

a.

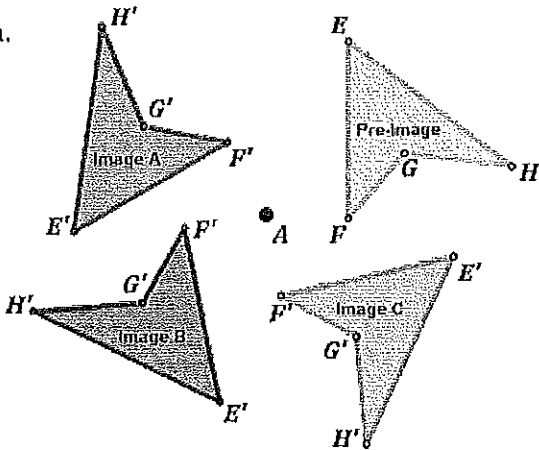


b.

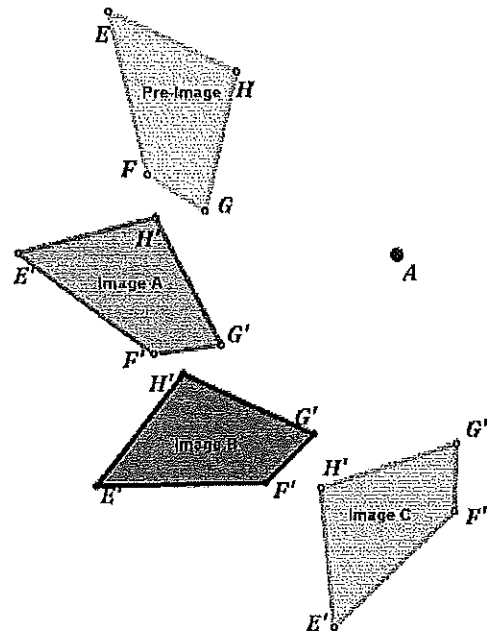


8. Which Image is a rotation of 120° about the point A?

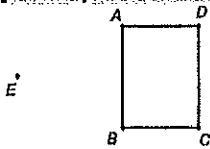
a.



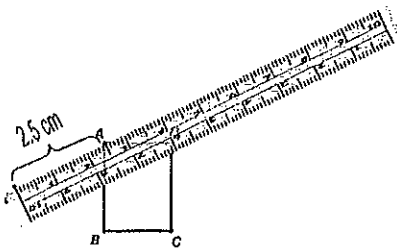
b.



[Example Dilation]: Dilate the $\square ABCD$ by a factor of 2.0 from point E.

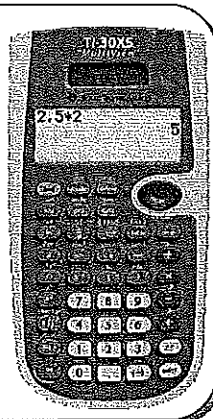


Step 1: Measure the distance from the point of dilation to a point to be dilated (preferably using centimeters).

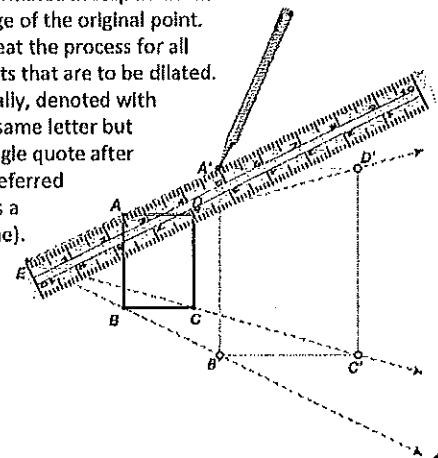


Step 2: Multiply the measured distance by the scale factor.

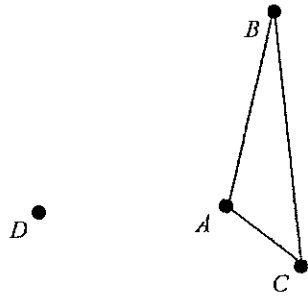
$$2.5\text{ cm} \times 2.0 = 5\text{ cm}$$



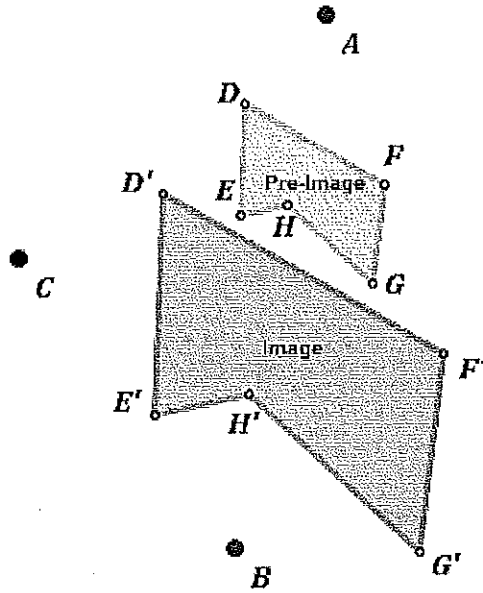
Step 3: With the ruler in the same place as it was in step #1, mark a point at the measured distance determined in step #2 as the image of the original point. Repeat the process for all points that are to be dilated. Usually, denoted with the same letter but a single quote after it (referred to as a prime).



9. Dilate the $\triangle ABC$ by a factor of $\frac{3}{2}$ from point D.

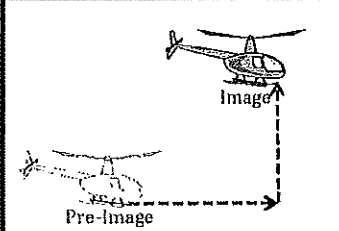
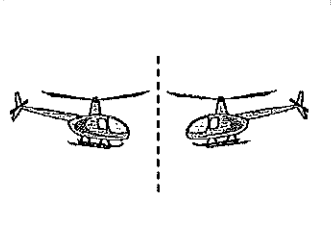
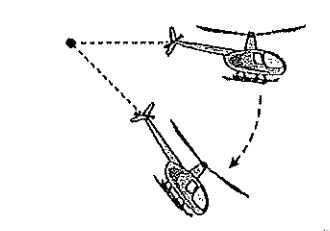
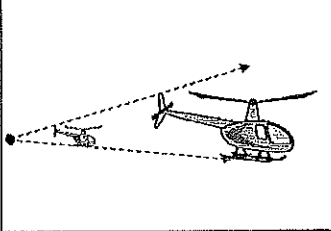


10. Which is the correct point of Dilation if the pre-image was dilated by a factor of 2?



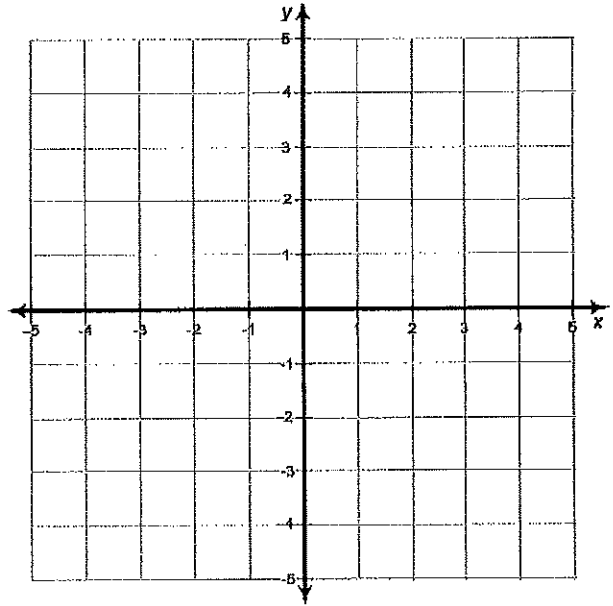
11. Which of the four transformations are isometries?

Transformation Types:

Translation ↓ Translation	Reflection ↓ Reflection	Rotation ↓ Rotation	Dilation ↓ Dilation
			

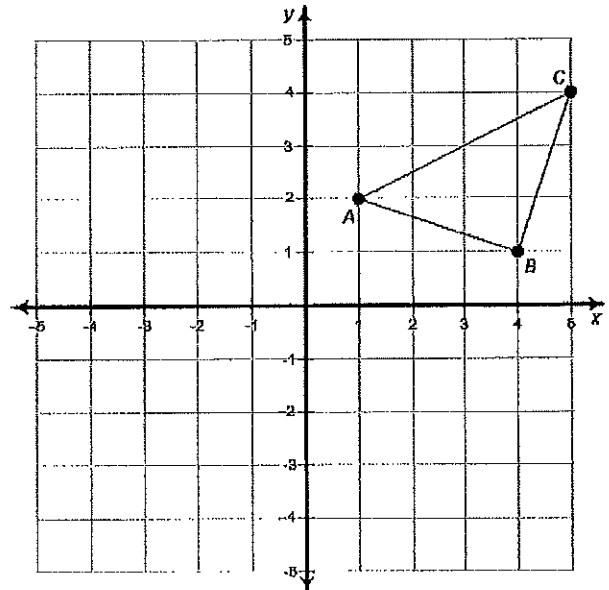
Translations

- Plot the points and create the pre-image triangle with vertices $M(-2, 3)$, $N(-3, 5)$, and $P(-5, 2)$. Then, determine the coordinates M' , N' , and P' of the image triangle that has been translated right 6 and down 3. Explain what you did to each of the coordinates.



Reflections

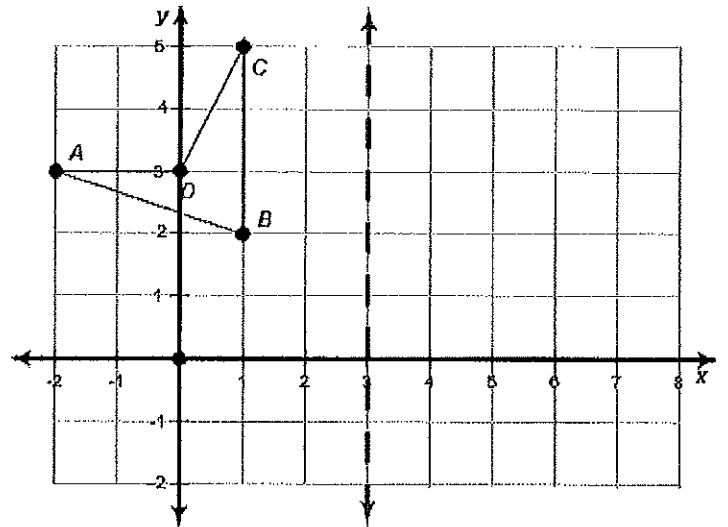
- List the coordinates of the triangle ABC
- Reflect the triangle ABC over the **y-axis** and list the coordinates of the vertex A' , B' , and C' . Describe what happened to the coordinates from the pre-image to the image.
- Reflect the triangle ABC over the **x-axis** and list the coordinates of the vertex A' , B' , and C' . Describe what happened to the coordinates from the pre-image to the image.



Reflections

5. List the coordinates of the quadrilateral ABCD

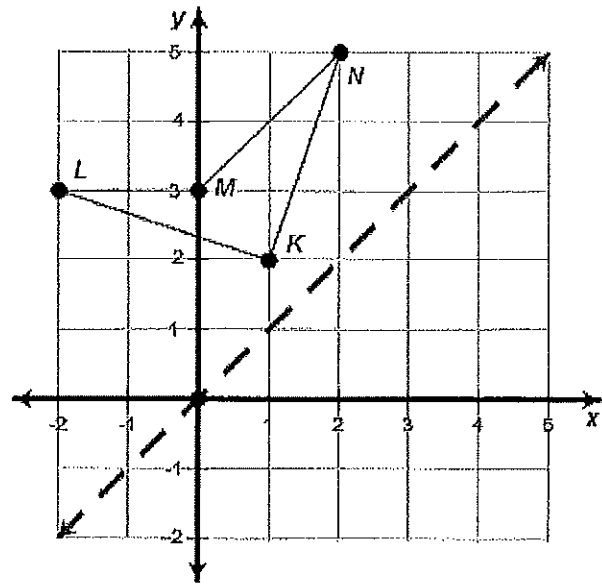
6. Reflect the quadrilateral ABCD over the line $x = 3$ and list the coordinates of the vertex A' , B' , C' , and D' .



Reflections

7. List the coordinates of the quadrilateral KLMN

8. Reflect the quadrilateral KLMN over the line $y = x$ and list the coordinates of the vertex K' , L' , M' , and N' . Describe what happened to the coordinates from the pre-image to the image.



Reflections

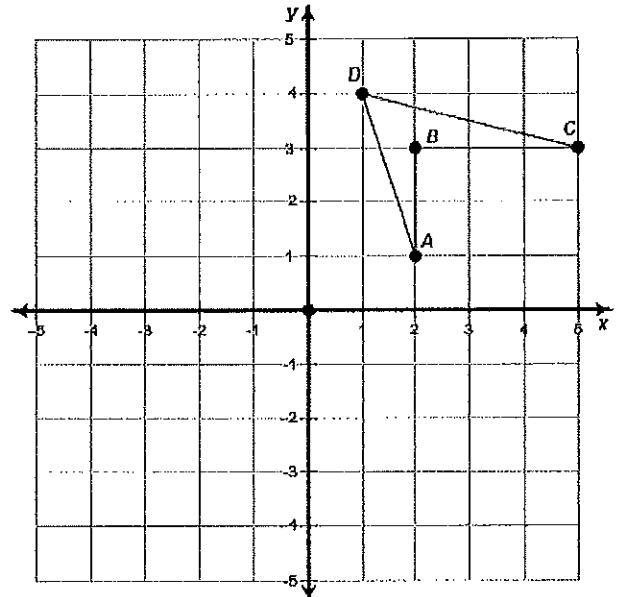
9. If the point A is located at $(-3, 2)$ and A' is the image of A after being reflected over the x-axis, what are the coordinates of A' ?

10. If the point B is located at $(-4, -1)$ and B' is the image of B after being reflected over the y-axis, what are the coordinates of B' ?

11. If the point C is located at $(2, -3)$ and C' is the image of C after being reflected over the line $y = x$, what are the coordinates of C' ?

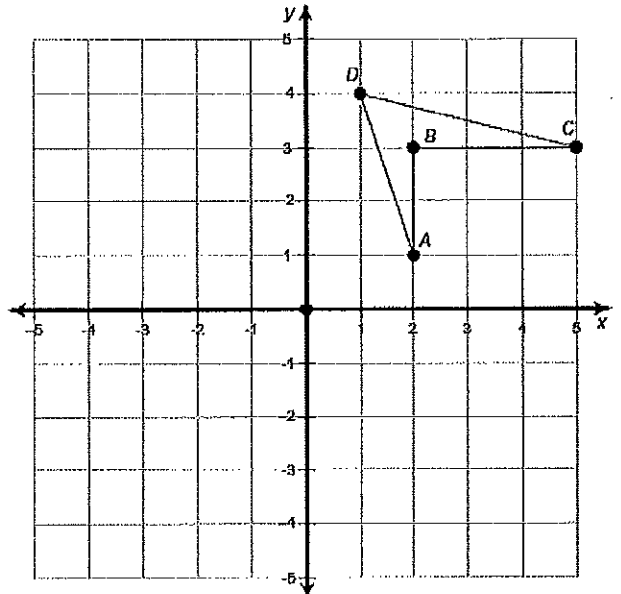
Rotations

12. List the coordinates of the quadrilateral ABCD.



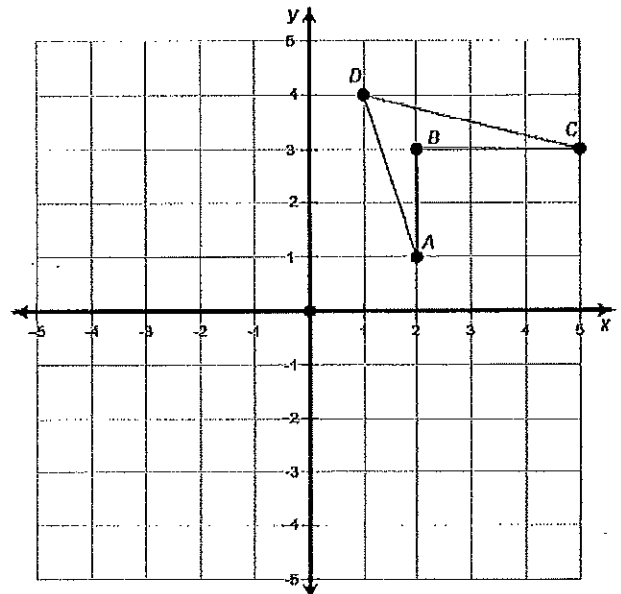
13. Rotate the quadrilateral ABCD about the origin by 90° and list the coordinates of the vertex A' , B' , C' , and D' . Describe what happened to the coordinates from the pre-image to the image.

14. Rotate the quadrilateral ABCD about the origin by 180° and list the coordinates of the vertex A' , B' , C' , and D' . Describe what happened to the coordinates from the pre-image to the image.



15. Rotate the quadrilateral ABCD about the origin by 270° and list the coordinates of the vertex A' , B' , C' , and D' . Describe what happened to the coordinates from the pre-image to the image.

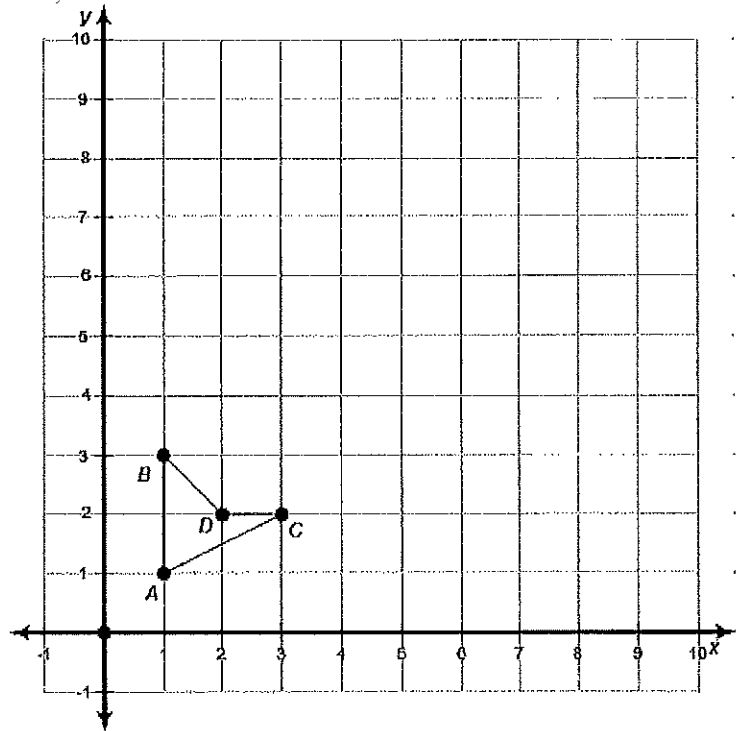
16. If the point A is located at $(-3, 2)$ and A' is the image of A after being rotated about the origin by 270° . What are the coordinates of A' ?



Dilations

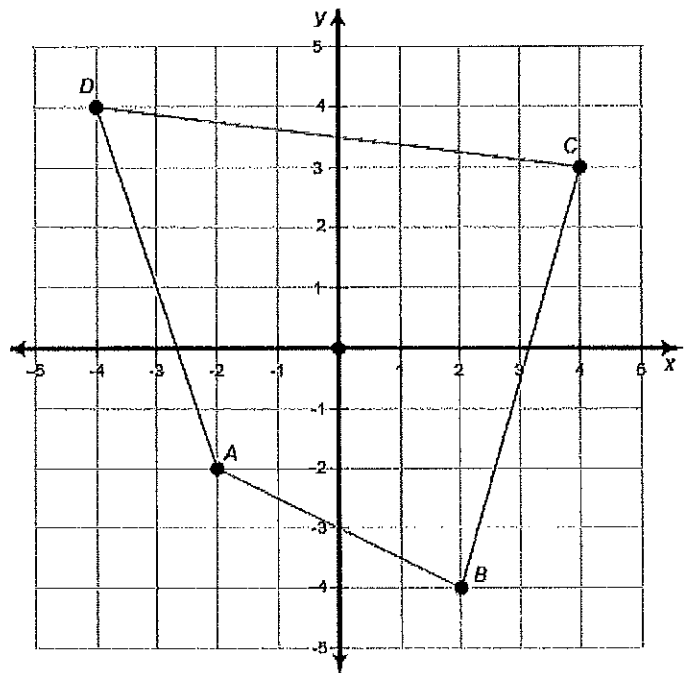
17. List the coordinates of the quadrilateral ABCD.

18. Dilate the quadrilateral ABCD **by a scale factor of 3 from the origin** and list the coordinates of the vertex A', B', C', and D'. Describe what happened to the coordinates from the pre-image to the image.



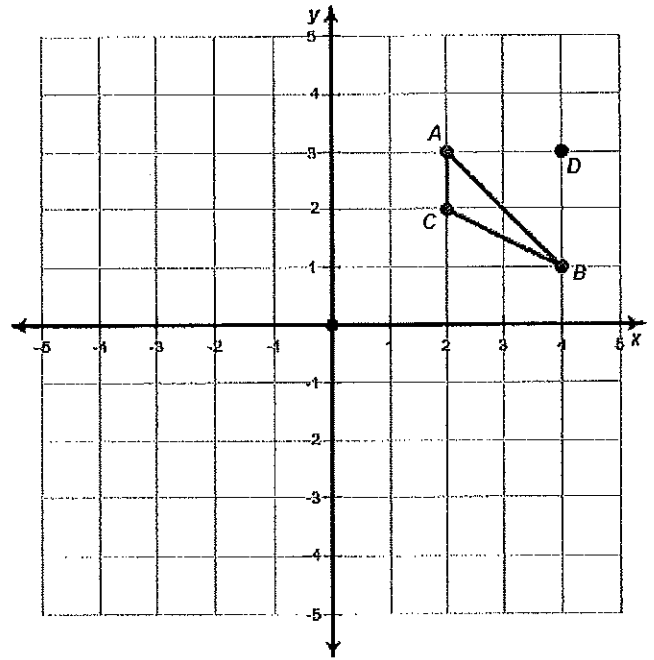
19. List the coordinates of the quadrilateral ABCD.

20. Dilate the quadrilateral ABCD **by a scale factor of $\frac{1}{2}$ from the origin** and list the coordinates of the vertex A', B', C', and D'. Describe what happened to the coordinates from the pre-image to the image.



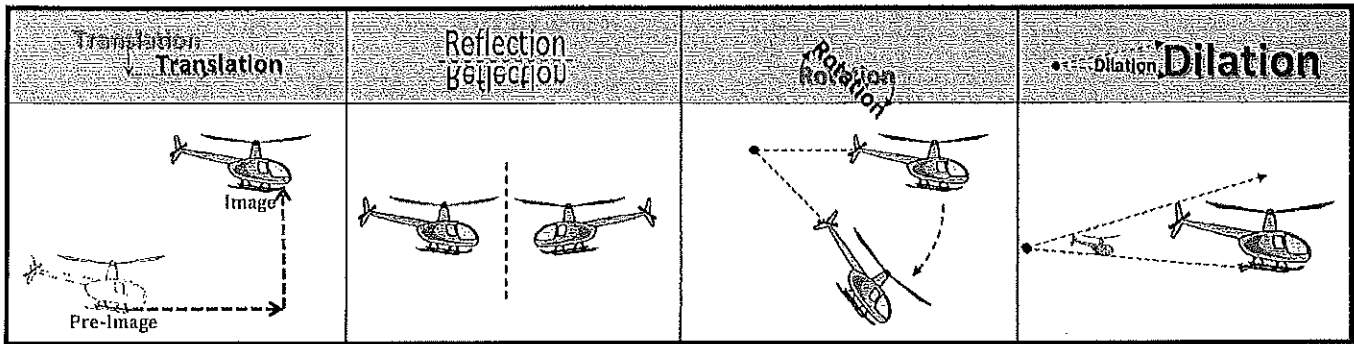
21. If the point A is located at (3, - 2) and A' is the image of A after being dilated by a scale factor of 5 from the origin. What are the coordinates of A'?

22. Dilate the triangle ABC by a scale factor of 4 from the point D and list the coordinates of the vertex A', B', and C'.



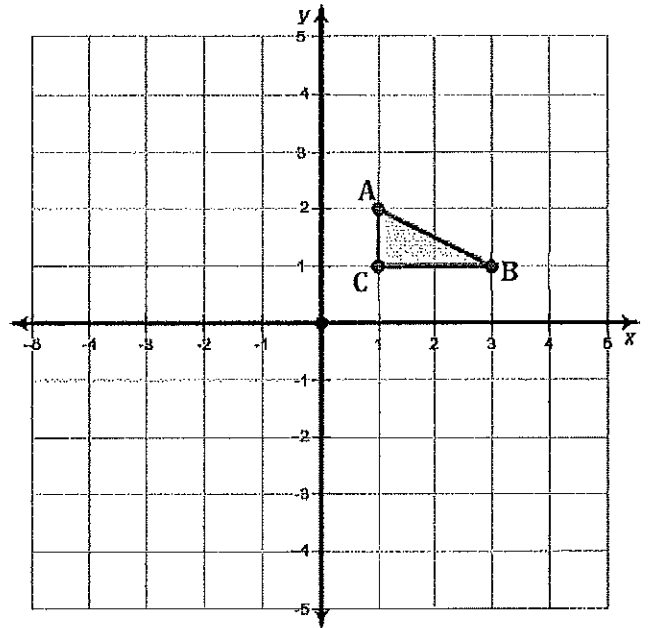
Create a list of all of the basic coordinate transformation rules:

Transformation Types:



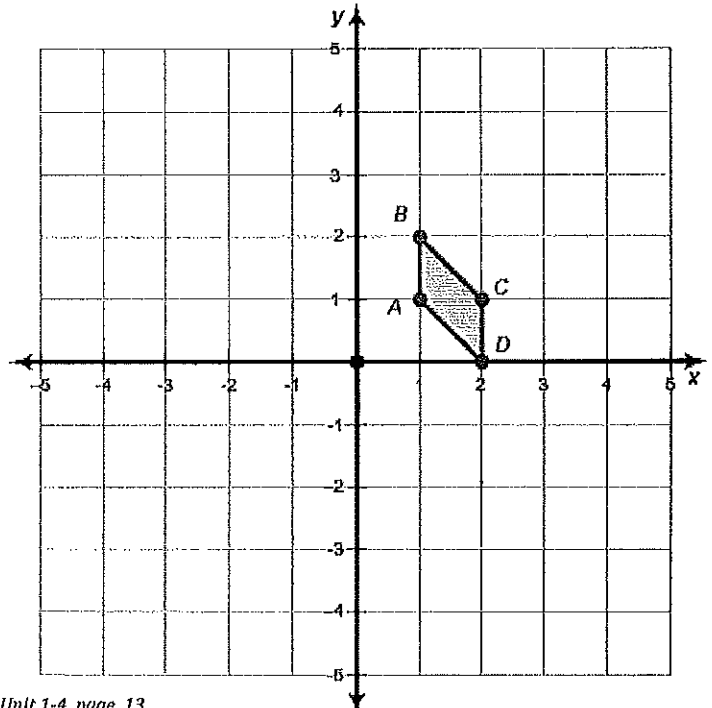
1. Consider the pre-image triangle with vertices $A(1,2)$, $B(3,1)$, and $C(1,1)$.

- Rotate the pre-image triangle ABC 90° about the origin and label this triangle $A'B'C'$
- Reflect the triangle $A'B'C'$ over the x -axis and label this triangle $A''B''C''$.

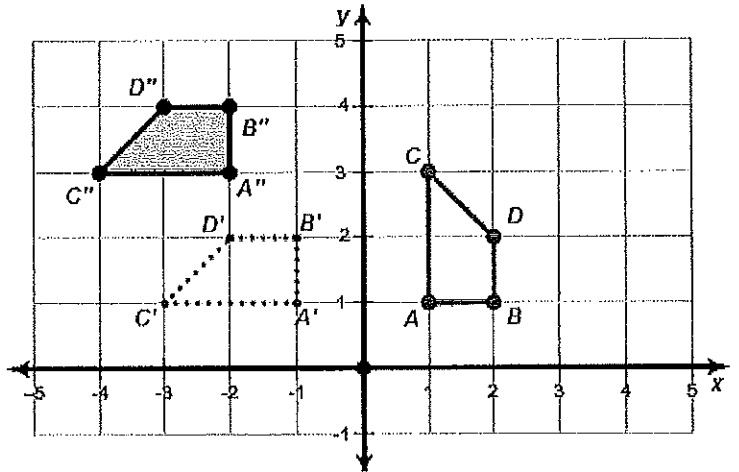


2. Consider the pre-image quadrilateral with vertices $A(1,1)$, $B(1,2)$, $C(2,1)$, and $D(2,0)$

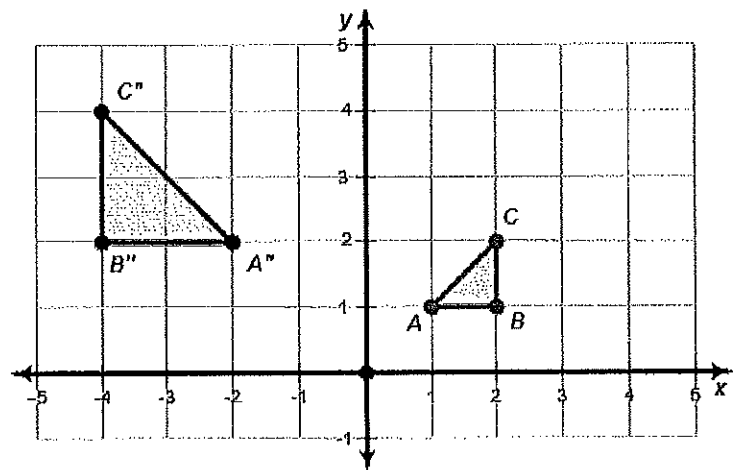
- Dilate the quadrilateral by a factor of 2 from the origin and label the image quadrilateral $A'B'C'D'$.
- Translate the quadrilateral image $A'B'C'D'$ down 5 units and left 1 unit. Label this new image $A''B''C''D''$.
- Reflect the quadrilateral image $A''B''C''D''$ over the y -axis and label this image $A'''B'''C'''D'''$



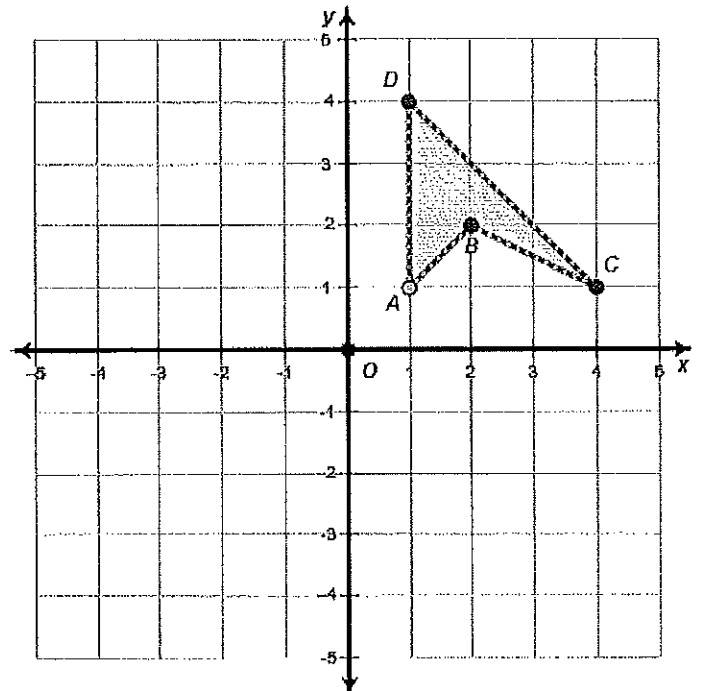
3. Describe two transformation that would map quadrilateral ABCD onto quadrilateral A''B''C''D''



4. Describe two transformation that would map triangle ABC onto triangle A''B''C''

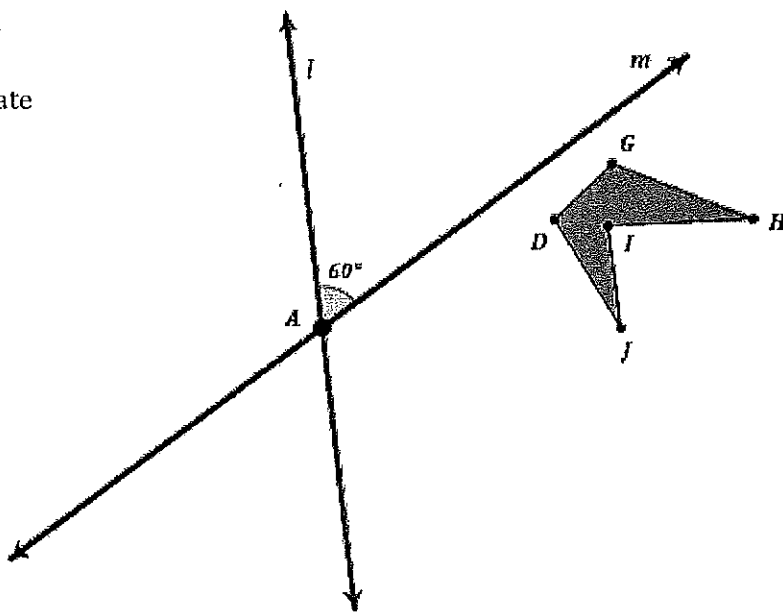


5. First reflect quadrilateral ABCD over the y-axis and label the triangle A'B'C'D'. Then, reflect quadrilateral A'B'C'D' over the x-axis and label this quadrilateral A''B''C''D''.



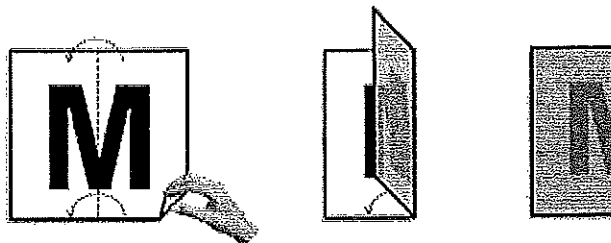
6. Anytime you use a double reflection, there should be a rotation about the intersection of the reflection lines that maps the pre-image onto the final image. In this example what is the amount of the rotation?

7. Given that pentagon $DGHIJ$ is first reflected over **line m** to create the image $D'G'H'I'J'$. Then, the image $D'G'H'I'J'$ is reflected over the **line l** to create the image $D''G''H''I''J''$. What is a different transformation that would also map $DGHIJ$ onto $D''G''H''I''J''$?

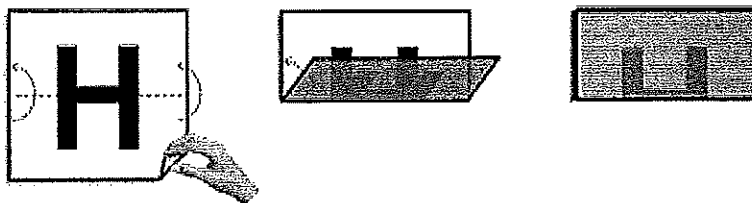


Symmetries

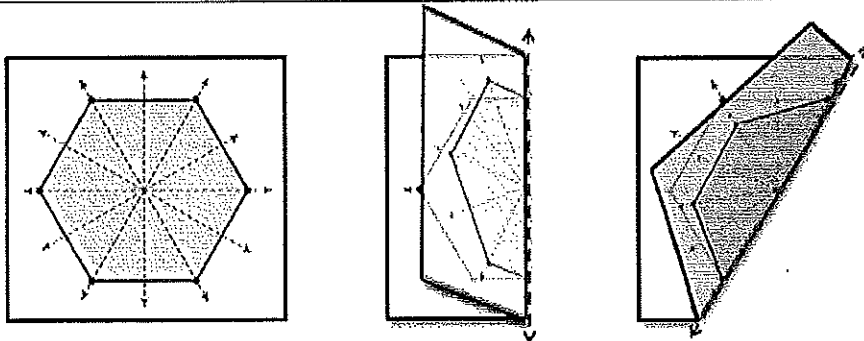
A shape has a **vertical line of symmetry** if you can fold it in half vertically and have the halves match up. On a scrap sheet of paper try folding the letter "M" in half.



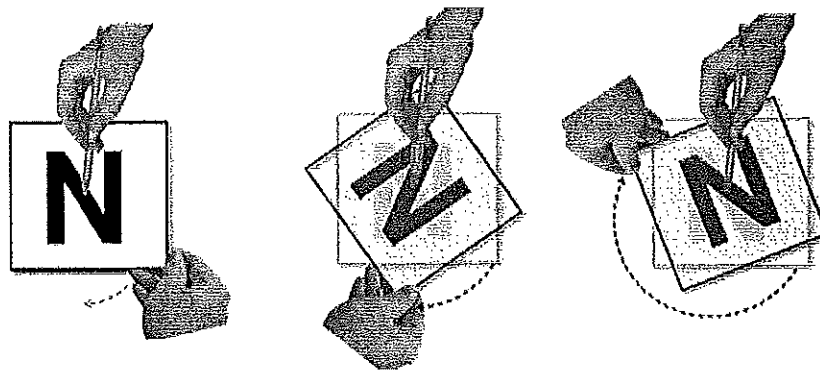
A shape has a **horizontal line of symmetry** if you can fold it in half horizontally and have the halves match up. On a scrap sheet of paper try folding the letter "H" in half.



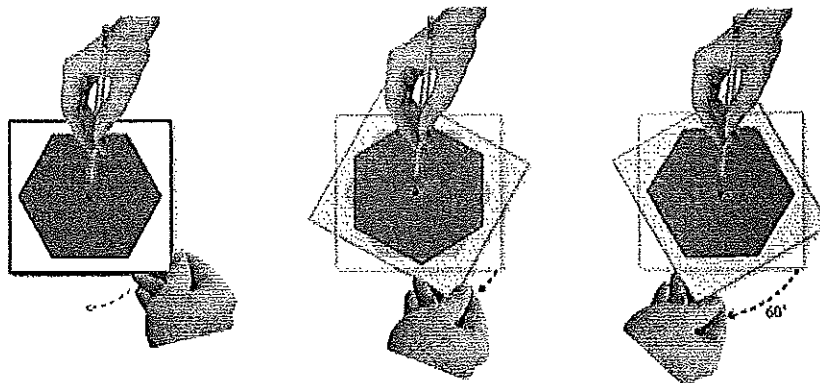
A general **line of symmetry** is any line in which you can fold the shape in half and it maps onto itself. Consider the hexagon at the right has 6 lines of symmetry.



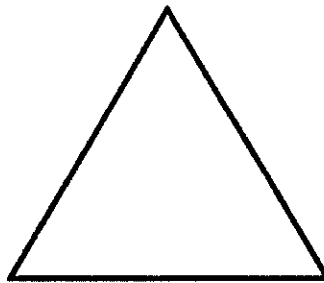
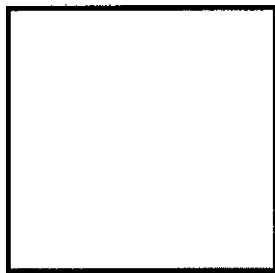
A shape has **point symmetry** if you can rotate the shape 180° from a center point and have it look the same as it did before you rotated it. On a scrap sheet of paper try rotating a letter "N" as shown at the right.



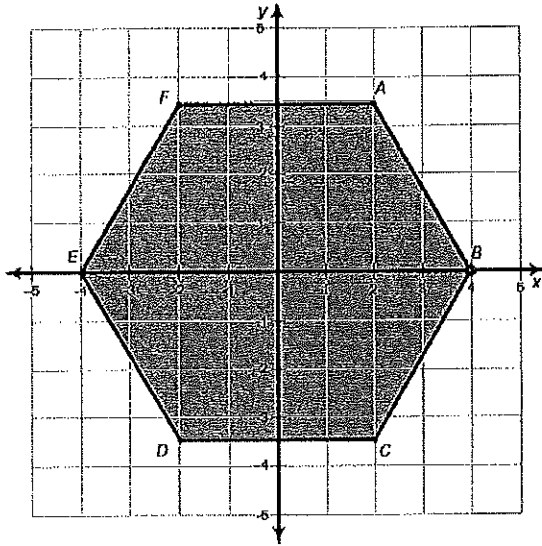
A shape has **rotational symmetry** if you can rotate the shape by a set degree from a center point and have it look the same as it did before you rotated it. On a scrap sheet of paper try rotating a hexagon (which has 60° rotational symmetry.)



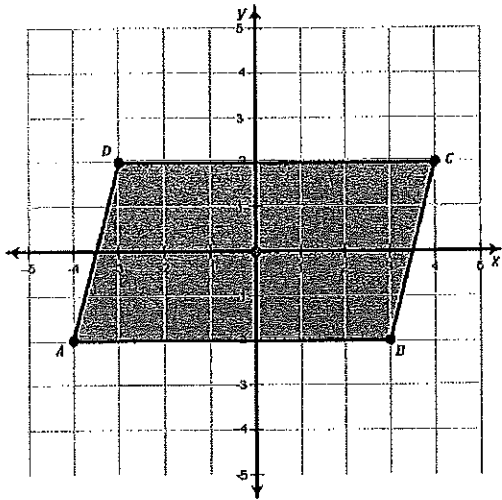
1. Provide 2 Letters of the Alphabet that have a vertical line of symmetry.
2. Provide 2 Letters of the Alphabet that have a horizontal line of symmetry.
3. Provide 2 Letters of the Alphabet that have both a horizontal line of symmetry and vertical line of symmetry.
4. Provide a letter of the Alphabet that has point symmetry but NOT a vertical line of symmetry.
5. Which letter depending on how it's written could have infinite lines of symmetry?
6. Draw all the lines of symmetry for the following regular shapes.



7. Describe in detail at least three transformations that would map hexagon ABCDEF onto itself.



8. Describe any symmetries that parallelogram ABCD might have.

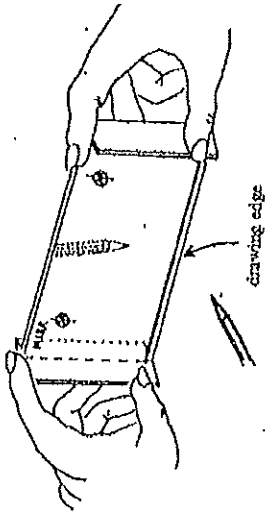


9. Ambigrams are usually words created with point symmetry.



MIRA ACTIVITIES

Hold the mira on your desk as shown in the illustration. There is a "right" and a "wrong" side for looking into the mira. If you can read the words at the top of the mira you are looking into the "right" side or face of the mira. In this position the mira's *drawing edge* will be touching the desk and will also be facing you. Notice that the drawing edge is set back a little behind the face of the mira.



If the mira rocks on your desk in this position then the desk is not flat. Move the mira about until you find a section of flat surface where rocking does not occur. Use this section in all of the following activities.

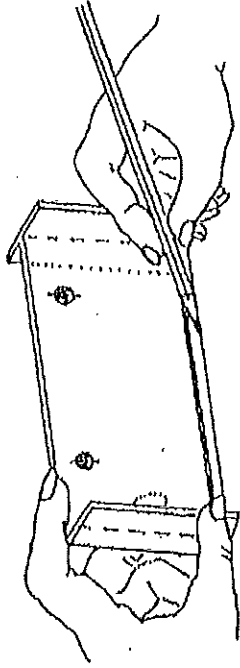
Place your mira on this page so that its drawing edge lies along the line m . Read the "hidden" message below.

sssd ussksk z fr or usskc oust the cscps at the top of the bckdr
 .surdv' Optrvks the surks of the zmsk crks at the pogrsk of pss bckb
 Now, move the mira so that you can see the words below in the reflection
 What you are reading is called the *reflection image* of the message below
 looking into the face of the mira
 If you can't see the words in the reflection, don't worry, you're just

If you have followed the instructions of this activity, you should now be looking into the face of the mira.

What you are reading is called the *reflection image* of the message below.

Now move the mira about and observe what happens to the reflection image. Observe the image of the small circle at the bottom of this page and make it fit or map onto the circle at the top of the page.



In the illustration a student is drawing a Mira line. Notice how he holds the Mira steady with one hand and traces along the drawing edge with the other. This tracing is called a Mira line.

Adjust the position of your Mira so that the image of the point P maps onto the point Q. Is the drawing edge face you? Hold the Mira firmly in this position and trace along the drawing edge. Be sure to use a sharp pencil.

Q is said to be the "image of P" of P is reflected onto Q."

Locate the image Y of X in the Mira line you have drawn.

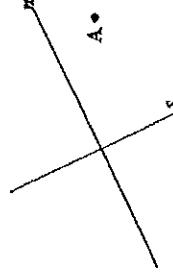
Q •

X •

P •

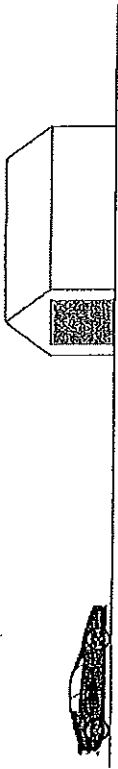
Place the drawing edge of your Mira along n . Notice that s is reflected onto itself. Lines n and s are perpendicular ($n \perp s$).

Locate a point called A' that is the Mira image of A over n ; then draw the line $\overline{AA'}$. How are $\overline{AA'}$ and n related? How are $\overline{AA'}$ and s related?

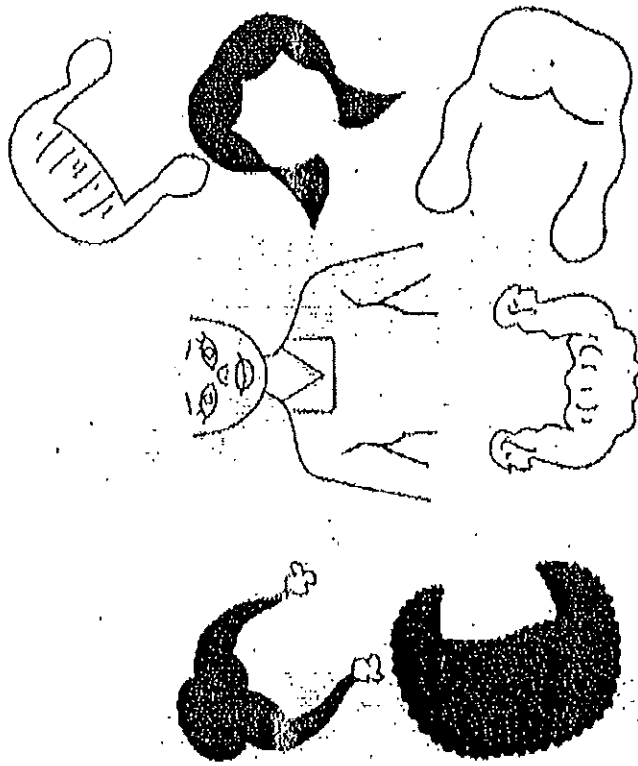


MIRA FUN

1. Use the mira to park the car into the garage.



2. Find the 6 mira lines to place each wig on the girl's head.



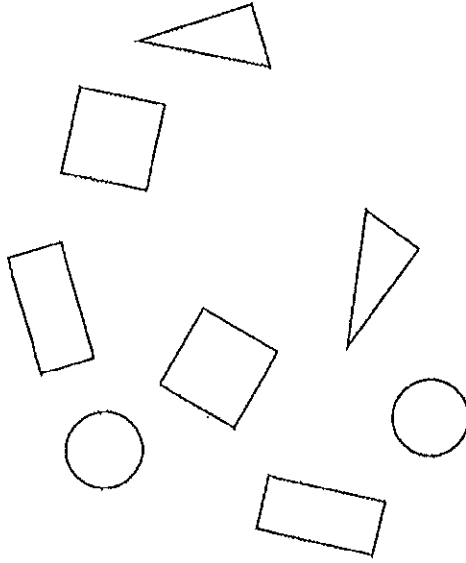
Reflections of Shapes

Place your mira between the two circles below and adjust it so that the reflection image of one circle coincides with the other circle. Draw the Mira line.

Try this with the two triangles.

Reflect one square onto the other.

Can you do the same thing with the two rectangles?

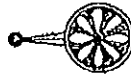
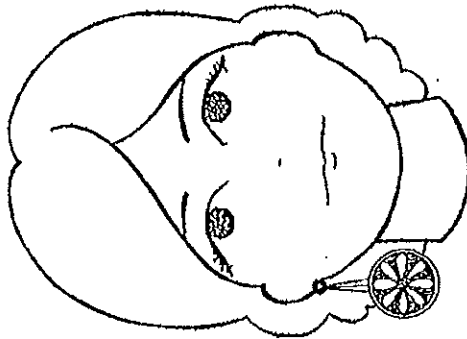


Name _____ Date _____

Worksheet 1

Introduction to the Mira (continued)

2. Using the Mira, put the earring on the woman. Then reach behind the Mira and trace the earring on the woman.

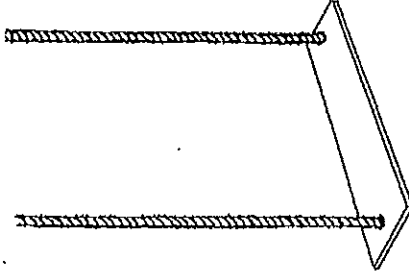
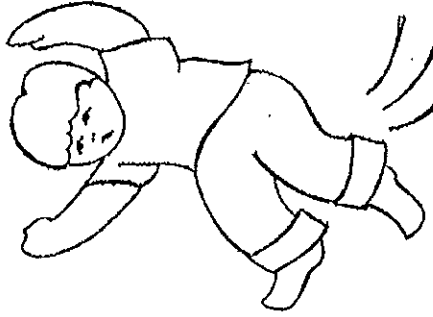


Name _____ Date _____

Worksheet 1

Introduction to the Mira

1. Use the Mira to put the child on the swing. Then reach behind the Mira and trace the child on the swing.



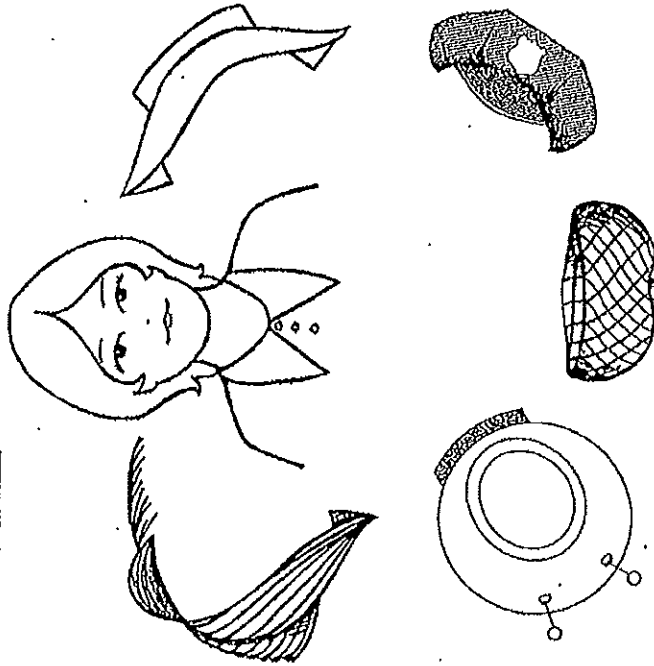
Name _____

Date _____

Worksheet 1

Introduction to the Mira (continued)

3. Using the Mira, put each hat on the woman. Select the hat you like best and trace it on the woman.



Name _____

Date _____

Worksheet 1

Introduction to the Mira (continued)

4. Using the Mira, put each hat (collar, cap) on the man. Select the hat you like best and trace it on the man.

